En el espacio vectorial euclideo $\mathbb{R}^4$ sobre el cuerpo de los números reales con el producto escalar usual se considera el siguiente subconjunto:

$$U = \{(x, y, z, t) \,/\, y + t = -z\}$$

Demostrar que U es un subespacio vectorial de $\mathbb{R}^4$. Calcular una base, dimensión y ecuaciones paramétricas de U. ¿Es la base obtenida de U ortogonal? En caso negativo, obtén una base ortogonal de U.

$$\left.\begin{array}{l}(x_1, y_1, z_1, t_1) \\ (x_2, y_2, z_2, t_2)\end{array}\right\} \in U \quad \begin{array}{l} \longrightarrow y_1 + t_1 = -z_1 \\ \longrightarrow y_2 + t_2 = -z_2 \end{array}$$

$$\alpha, \beta \in \mathbb{R}$$

$$\alpha(x_1, y_1, z_1, t_1) + \beta(x_2, y_2, z_2, t_2) \overset{??}{\in} U$$

$$\|$$

$$(\alpha x_1, \alpha y_1, \alpha z_1, \alpha t_1) + (\beta x_2, \beta y_2, \beta z_2, \beta t_2)$$

$$\|$$

$$(\alpha x_1 + \beta x_2, \underline{\alpha y_1 + \beta y_2}, \alpha z_1 + \beta z_2, \underline{\alpha t_1 + \beta t_2})$$

$$(\alpha y_1 + \beta y_2) + (\alpha t_1 + \beta t_2) = \alpha(y_1 + t_1) + \beta(y_2 + t_2)$$

$$\|$$

$$\alpha(-z_1) + \beta(-z_2)$$

$$-(\alpha z_1 + \beta z_2) \checkmark$$

$$y + t = -z \Rightarrow y + z + t = 0$$

$$\left.\begin{array}{l} x = \mu \\ y = -\alpha - \beta \\ z = \alpha \\ t = \beta \end{array}\right\} \text{Ec. param.}$$

$$\dim U = 3$$

$$B_U = \{\underset{u_1}{(1,0,0,0)}, \underset{u_2}{(0,-1,1,0)}, \underset{u_3}{(0,-1,0,1)}\}$$

$$\langle u_2, u_3 \rangle = \langle (0,-1,1,0), (0,-1,0,1) \rangle =$$

$$= 1 \neq 0 \Rightarrow u_2 \not\perp u_3$$

<u>Gram - Schmidt</u> $\quad B_{ortg} = \{e_1, e_2, e_3\}$

$$e_1 = u_1 = \boxed{(1,0,0,0)}$$

$$e_2 = u_2 - \frac{\langle u_2, u_1 \rangle}{\|u_1\|^2} \cdot u_1 = (0,-1,1,0) -$$

$$- \frac{\langle (0,-1,1,0), (1,0,0,0) \rangle}{\langle (1,0,0,0), (1,0,0,0) \rangle} (1,0,0,0) =$$

$$= (0,-1,1,0) - \frac{0}{1} (1,0,0,0) =$$

$$= \boxed{(0,-1,1,0)}$$

$$e_3 = u_3 - \cancelto{0}{\frac{\langle u_3, u_1 \rangle}{\|u_1\|^2}} \cdot u_1 - \frac{\langle u_3, u_2 \rangle}{\|u_2\|^2} \cdot u_2$$

$$\langle u_3, u_1 \rangle = \langle (0,-1,0,1), (1,0,0,0) \rangle = 0$$

$$\langle u_3, u_2 \rangle = 1$$

$$\langle u_2, u_2 \rangle = \langle (0,-1,1,0), (0,-1,1,0) \rangle =$$

$$= 1 + 1 = 2$$

$$e_3 = (0,-1,0,1) - \frac{1}{2} (0,-1,1,0) =$$

$$= \boxed{\left(0, -\frac{1}{2}, -\frac{1}{2}, 1\right)}$$